Exact Sampling Distribution

Student's 't' Distribution

- Initial assumption is that there is no significant difference between the sample mean $\overline{x}$ and the population mean M.
- Find t using formula
- If calculated $|t|>t_0$, tabulated t, then the hypothesis is rejected & if calculated $|t|<t_0$, then hypothesis may be accepted at the level of significance adopted.

NOTE: degrees of freedom = n – 1 . Also keep a watch on if it's a 1 – tail or 2 – tail test.

Assumptions for Student's t – test:

- The parent population from which the samples is drawn is normal
- The sample observations are independent, i.e., the sample is random
- The population standard deviation σ is unknown.

(1) A machinist is making engine parts with axle diameters of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a S.D. of 0.040 inch. Compute the statistic you would use to test whether the work is meeting the specifications. [Ans: t=3.15;]

(2) The mean weekly sale of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 & showed a S.D. of 17.2. Was the advertising campaign successful? [Ans: t=1.97; t_0=1.72; Successful]

(3) A random sample of 10 boys had the following I.Q.'s: 70,120,110,101,88,83,95,98,107,100. Do these data support the assumption of a population mean I.Q. of 100? Find a reasonable range in which most of the mean I.Q. values of the samples of 10 boys lie. [Ans; $|t|$=0.62; t_0=2.262; data is consistent with the assumption; [86.99,107.41]]

(4) The heights of 10 males of a given locality are found to be 70,67,62,68,61,68,70,64,64,66 inches. Is it reasonable to believe that the average height is greater than 64 inches? Test at 5% significance level.

(5) A random sample of 16 values from a normal population showed a mean of 41.5 inches & the sum of squares of deviations from this mean equal to 135 square inches. Show that the assumption of a mean of 43.5 inches for the population is not reasonable. Obtain 95% & 99% fiducial limits for the same. Given $v=15$, $P=0.05$, $t= 2.131$ & $P=0.01$, $t=2.947$.

(6) Below are given the gain in weights (in kgs) of pigs fed on two diets A & B.

diet A: 25,32,30,34,24,14,32,24,30,31,35,25

diet B:44,34,22,10,47,31,40,30,32,35,18,21,35,29,22

Test, if the two diets differ significantly as regards their effect on increase in weight.

(7) Sample of two types of electric light bulbs were tested for length of life & following data were obtained:

	Type I	Type II
Sample No	$n_1=8$	$n_2=7$
Sample Means	1234 hrs.	1036hrs
Sample S.D.'s	36 hrs	40 hrs

Is the difference in the means sufficient to warrant that type I is superior to type II regarding length of life?

(8) The height of six randomly chosen sailors are (in inches):63,65,68,69,71,72. Those of 10 randomly chosen soldiers are 61,62,65,66,69,69,70,71,72,73. Discuss, the light that these data throw on the suggestion that sailors are on the average taller than the soldiers.

(9) To test the claim that the resistance of electric wire can be reduced by at least 0.05 ohm by alloying, 25 values obtained for each alloyed wire & standard wire produced the following results:

	Mean	Standard deviation
Alloyed wire	0.083 ohm	0.003 ohm
Standard wire	0.136 ohm	0.002 ohm

Test at 5% level whether or not the claim is substantial.

(10) A certain stimulus administered to each of the 12 patients resulted in the following increase of blood pressure:

5,2,8,-1,3,0,-2,1,5,0,4,6

Can it be concluded that the stimulus will, in general, be accompanied by an increase in blood pressure?

(11) In a certain experiment to compare two types of animal foods A & B, the following results of increase in weights were observed in animals:

	1	2	3	4	5	6	7	8	Total
Food A	49	53	51	52	47	50	52	53	407
Food B	52	55	52	53	50	54	54	53	423

(i) Assuming that the two samples of animals are independent, can we conclude that food B is better than food A?

(ii) Also examine the case when the same set of eight animals were used in both the foods.

(12) Two laboratories carry out independent estimates of particular chemicals in a medicine produced by a certain firm. A sample is taken from each batch, halved & the separate halves sent to the two laboratories. The following data is obtained:

No. of samples	10
Mean value of the difference of estimates	0.6
Sum of the squares of the difference (from their means)	20

Is the difference significant? (value of t at 5% level for 9 d.f. is 2.262)

Chi - Square (X^2) distribution

(1) The demand for a particular square part in a factory was found to vary from day – to – day. In a sample study the following information was obtained:

Days	Mon	Tue	Wed	Thursday	Fri	Sat
No. of parts demanded	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week. (Given the values of chi-square significance at 5,6,7 d.f. are respectively 11.07, 12.59, 14.07 at 5% level of significance.)

(2) The following figures show the distribution of digits in numbers chosen at random from a telephone directory:

Digits:	0	1	2	3	4	5	6	7	8	9	Total
frequency:	1026	1107	997	966	1075	933	1107	972	964	853	10,000

Test whether the digits may be taken to occur equally frequently in the directory.

(3) A sample analysis of examination results of 200 MBA's was made. It was found that 46 students had failed, 68 secured a third division, 62 secured a second division & the rest were placed in first division. Are these figures commensurate with the general examination result which is in the ratio of 4:3:2:1 for various categories respectively?

(4) A survey of 800 families with four children each revealed the following distribution:

No. of boys:	0	1	2	3	4
No. of girls:	4	3	2	1	0
No. of families:	32	178	290	236	64

Is this result consistent with the hypothesis that male & female births are equally probable?

(5) When the first proof of 392 pages of a book of 1200 pages were read, the distribution of printing mistakes were found to be as follows:

No. of mistakes in a page (x):	0	1	2	3	4	5	6
No. of pages (f):	275	72	30	7	5	2	1

Fit a Poisson distribution to the above data & test the goodness of fit.

Test of Independence of Attributes – Contingency Table

1. Two sample polls of votes for two candidates A & B for a public office are taken, one from among the residents of rural areas. The results are given in the table below. Examine whether the nature of the area is related to voting preferences in this election.

Area	Votes for A	Votes for B
Rural	620	380
Urban	550	450

2. Out of 8000 graduates in a town 800 are females, out of 1600 graduate employees 120 are females. Use X^2 to determine if any distinction is made in appointment on the basis of sex. Value of X^2 at 5% level for one degree of freedom is 3.84.

3. A random sample of students of XYZ University was selected &asked their opinions about 'autonomous college'. The results are given below. The same number of each sex was included within each class – group. Test the hypothesis at 5% level that opinions are independent of the class groupings.

Class	Number favouring 'autonomous colleges'	Number opposed to 'autonomous colleges'
BA/BSc/Bcom Part I	120	80
BA/BSc/Bcom Part II	130	70
BA/BSc/Bcom Part III	70	30
MA/MSc/MCom	80	20

4. Two researchers adopted different sampling techniques while investigating the same group of students to find the number of students falling in different intelligence levels. The results are as follows:

Researcher	No. of students below Avg.	No. of students Avg.	No. of students above Avg.	No. of students Genius
X	86	60	44	10
Y	40	33	25	2

Would you say that the sampling technique adopted by the two researchers is significantly different? (Given 5% value of X^2 for d.f and d.f. are 5.991 and 7.82 respectively).